Nonlinear generation of eddy currents by crossed surface waves

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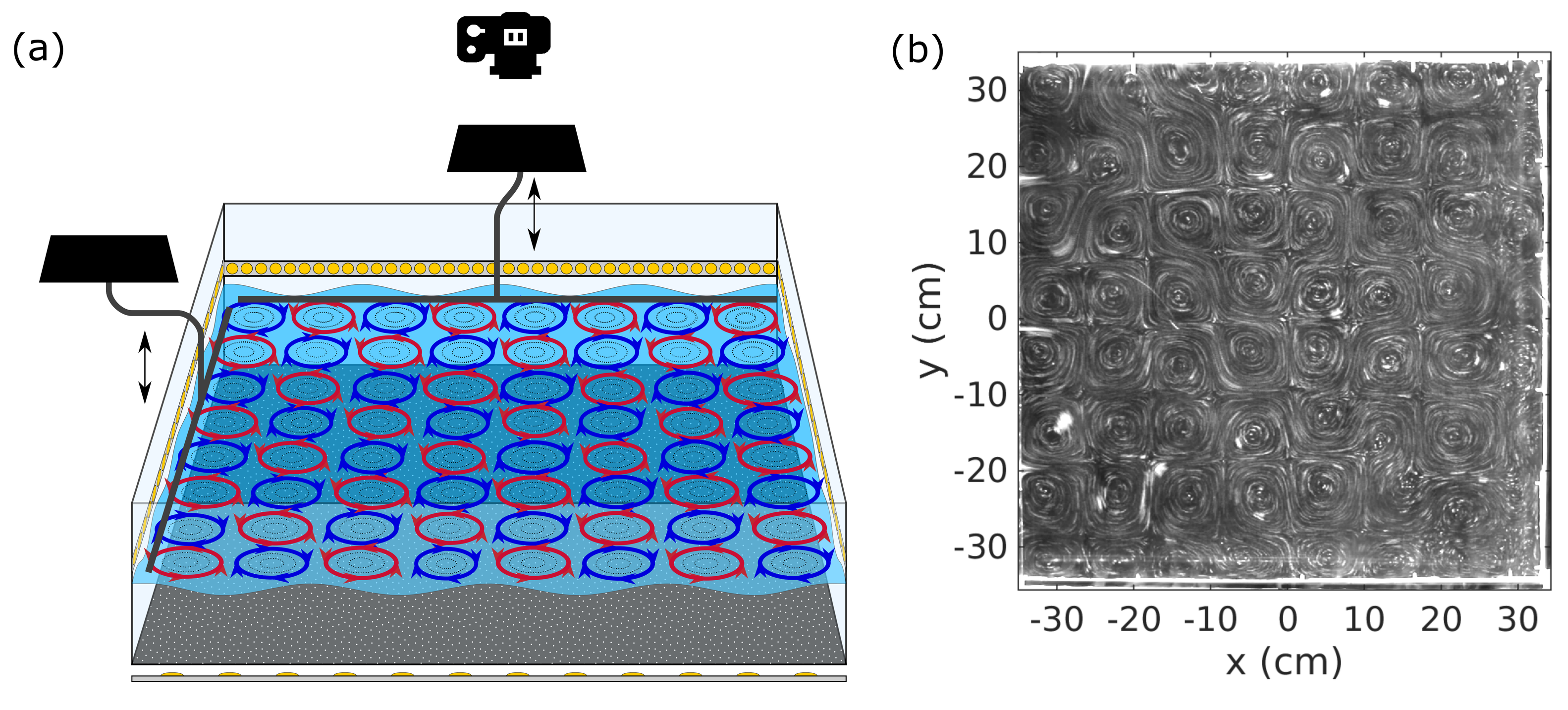
1. What is this poster about?

We study the mass-transport induced by crossed surface waves corresponding to a regular lattice of counter-rotating vortices. The Stokes drift describes the underlying flow, suggesting that surface waves are potential. However, in the experiments, the flow is much more intense. How is the energy of vertical wave oscillations injected into horizontal flow?

We show that an arbitrarily small viscosity breaks the potential approximation and substantially changes the generated flow. An effective force proportional to viscosity and quadratic in wave amplitudes excites an additional slow current in a thin viscous sublayer near the fluid surface. This current spreads into the fluid bulk due to viscosity and, surprisingly, in the stationary regime nothing depends on viscosity. The amplitude of the effective force is sensitive to the surface contamination and we demonstrate that a thin insoluble liquid film presented on the fluid surface can substantially increase the induced mass-transport.

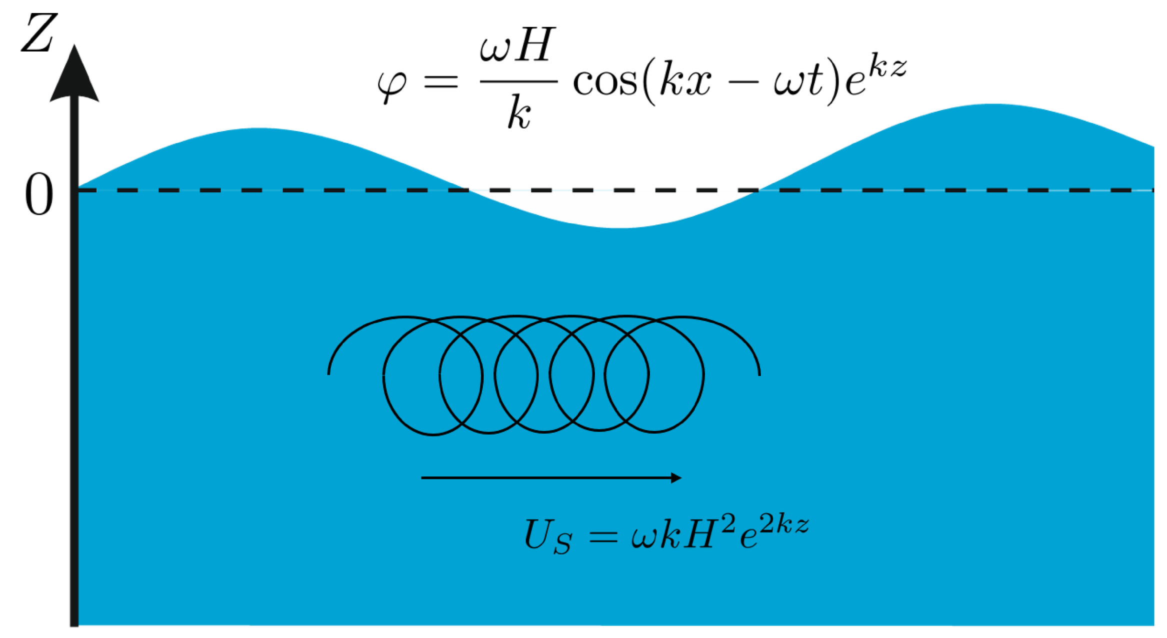


(a) Scheme of the experimental setup and (b) tracks of polyamide particles on the water surface.

2. Reminder: Stokes drift

The time derivative of the Lagrangian particle coordinate is defined by the fluid velocity $d\mathbf{R}/dt = \mathbf{v}(t,\mathbf{R})$. This nonlinear equation can be solved iteratively and the displacement of the particle is $\delta\mathbf{R} = \delta\mathbf{R}_1 + \delta\mathbf{R}_2 + \dots$, where $\delta R_{1i}(t) = \int_{t_0}^{t} dt'\, v_i(t',\mathbf{r}_0)$, $\delta R_{2i}(t) = \int_{t_0}^{t} dt'\, \partial_j v_i(t',\mathbf{r}_0)\delta R_{1j}(t')$. For a progressive wave in an inviscid fluid, George Stokes obtained:

$$\mathbf{V}_S = \langle \mathbf{v}(t,\mathbf{r}_0)\rangle + \langle \partial_j v_i(t,\mathbf{r}_0)\delta R_{1j}(t)\rangle = \omega k H^2 e^{2kz}.$$



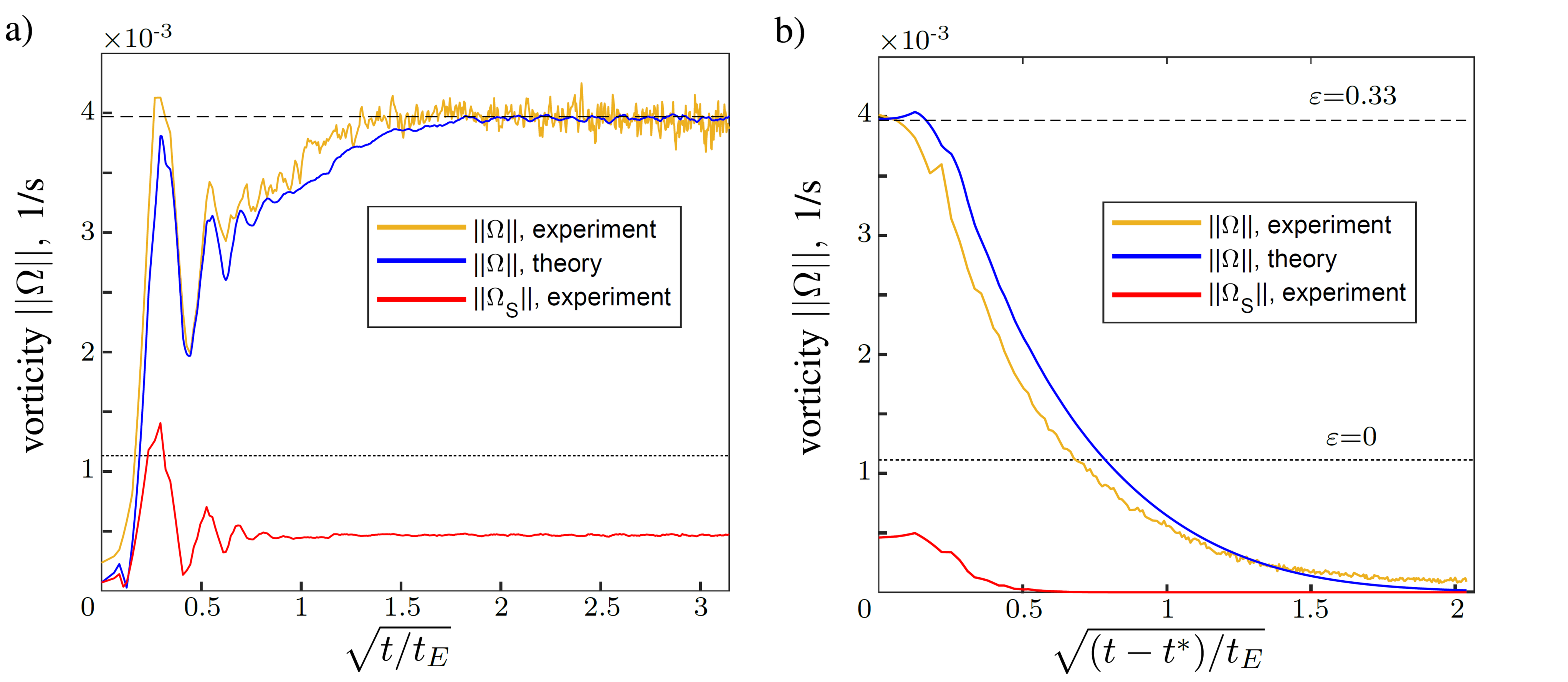
Schematic representation of the trajectories of fluid particles in a plane potential progressive wave.

In our experiments the surface elevation is $h(t,x,y) = H_1\cos(\omega t)\cos(kx) + H_2\cos(\omega t+\psi)\cos(ky)$ and we describe the corresponding mass-transport in terms of the vertical vorticity, $\Omega = \partial_x V_y - \partial_y V_x$, where V_x and V_y are horizontal components of the Lagrangian velocity of fluid particles. The contribution from the Stokes drift to the vertical vorticity is

$$\Omega_S = -\omega k^2 H_1 H_2 e^{2kz}\sin(kx)\sin(ky)\sin\psi.$$

3. Experimental Results

Formation (a) and decay (b) of eddy currents on the fluid surface for 38% glycerine-water solution, $t_E = 1/(2\nu k^2) \approx 125\,s$. The surface waves were excited at frequency of $3\,Hz$. Red curves demonstrate the Stokes drift contribution based on the experimentally measured wave amplitudes $H_{1,2}(t)$ and the phase difference $\psi(t)$.



- The Stokes drift contribution underestimates the intensity of eddy currents by about 8 times in the stationary regime.
- We are able to see eddy currents when the wave motion and the Stokes drift contribution have already disappeared. This observation proves the existence of Euler's contribution to the currents.

4. Why is fluid viscosity important?

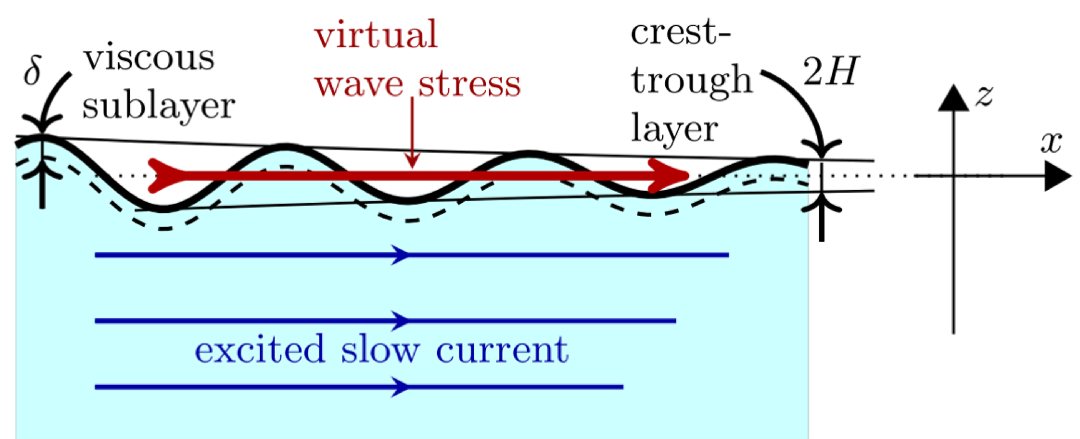
Surface wave possesses a momentum that is directed parallel to the direction of propagation and is proportional to the square of the wave amplitude:

$$\pi_x = \rho\left\langle \int_{-\infty}^{h} dz\, v_x \right\rangle = \rho \int_{-\infty}^{0} dz\, V_S = \frac{\rho\omega H^2}{2}.$$

Viscous dissipation leads to a decrease in the amplitude of the wave during its propagation, $H = H_0\exp(-4\gamma^2 kx)$, where $\gamma^2 = \nu k^2/\omega \ll 1$. It means that the momentum associated with the wave motion also decreases. Then conservation of the total momentum requires the presence of a force acting on the fluid. This force is applied in the viscous sublayer and it is of the second order in the wave amplitude and linear in the viscosity. One can assume that the force is tangent stress applied to the fluid surface (it is also known as the virtual wave stress):

$$\rho\nu\partial_z V_x|_{z=0} = F_x, \quad F_x = 2\rho\nu\omega(kH)^2.$$

The action of this force leads to the generation of a slow (second-order) current, which then spreads into the fluid bulk due to the viscosity.



5. Eulerian and Lagrangian Mean Flows

Evolution of the vertical vorticity $\Omega_E(x,y,z,t)$ in the Euler description is described by the diffusion equation

$$\partial_t\Omega_E - \nu\nabla^2\Omega_E = 0, \qquad z<0,$$

which has to be supplemented by a fixed-stress boundary condition at the surface,

$$\rho\nu\partial_z\Omega_E\big|_{z=0} = -F H_1(t)H_2(t)\omega k^2\sin(kx)\sin(ky)\sin\psi(t),$$

where $F = 2\rho\nu k$ characterizes the intensity of the applied virtual wave stress.

The induced Lagrangian mean flow corresponds to the sum of Euler and Stokes contributions to the mass-transport, $\Omega(t) = \Omega_E(t) + \Omega_S(t)$. In the stationary regime,

$$\Omega(x,y,z) = -\left[\sqrt{2}e^{kz\sqrt{2}} + e^{2kz}\right] H_1H_2\omega k^2\sin(kx)\sin(ky)\sin\psi.$$

- The experimental value for the surface flow is approximately 4 times greater than this answer.

6. Surface Contamination

For typical conditions the fluid surface is contaminated and we model this effect by a thin insoluble liquid film presented on the surface. We describe the film properties by the only parameter – the dilational elasticity $\varepsilon = -n_0\sigma'(n_0)k^2/(\rho\sqrt{\nu\omega^3}) > 0$, where n_0 is an equilibrium value of the film surface density.

The presence of surface film increases the virtual wave stress, and for the eddy currents we find:

$$\Omega(x,y,z) = -\left[\frac{\varepsilon^2 e^{kz\sqrt{2}}}{2\gamma(\varepsilon^2-\varepsilon\sqrt{2}+1)} + \sqrt{2}e^{kz\sqrt{2}} + e^{2kz}\right] H_1H_2\omega k^2\sin(kx)\sin(ky)\sin\psi.$$

Comparing this expression with experimental data, we obtain the compression modulus ε of the film.

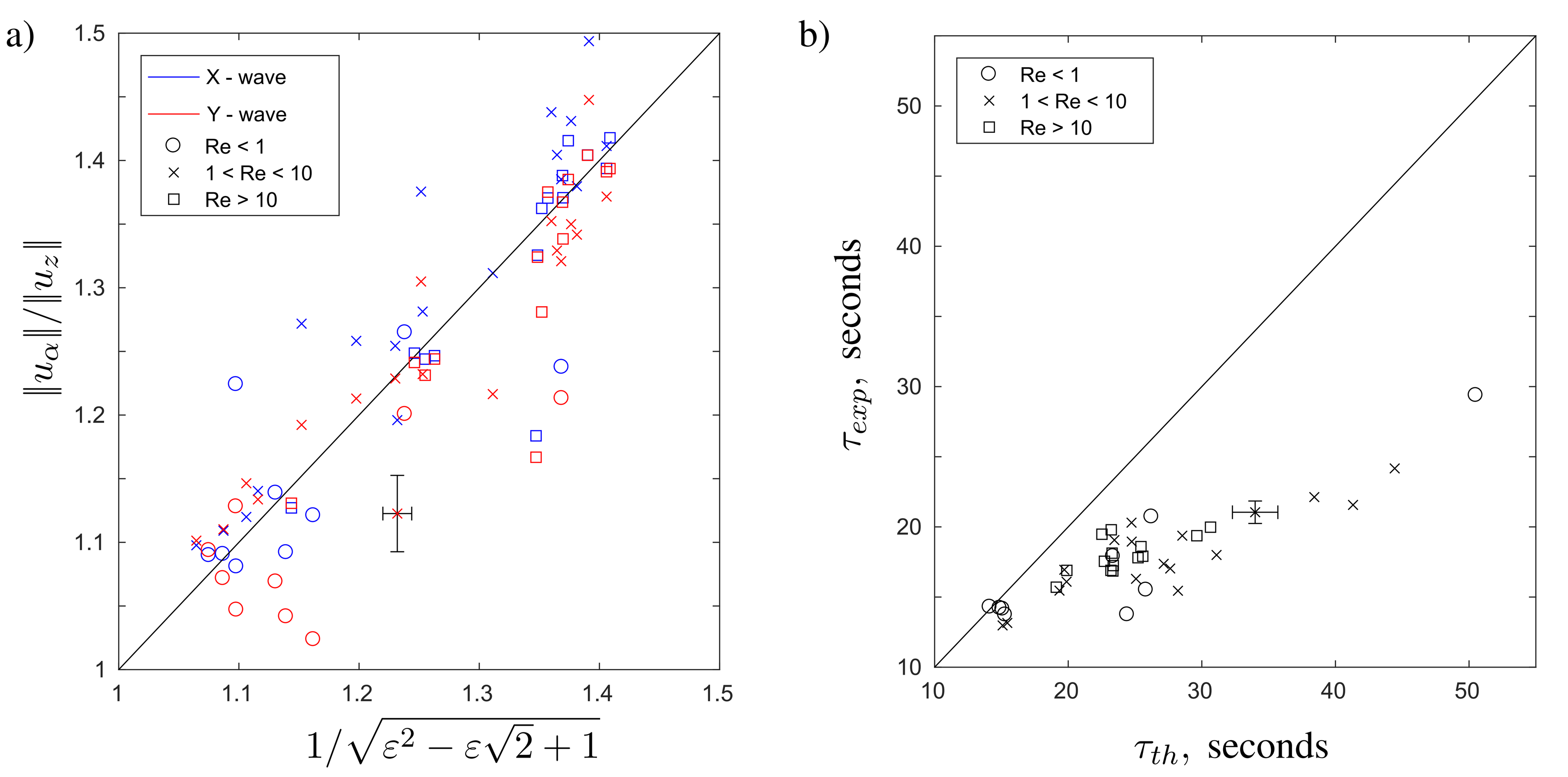
How to find out more?

- S. V. Filatov, V. M. Parfenyev, S. S. Vergeles, M. Yu. Brazhnikov, A. A. Levchenko, V. V. Lebedev, "Nonlinear generation of vorticity by surface waves", Phys. Rev. Lett. 116, 054501 (2016).
- V. M. Parfenyev, S. S. Vergeles, "Influence of a thin compressible liquid film on the eddy currents generated by interacting surface waves", Phys. Rev. Fluids 3, 064702 (2018).
- V. M. Parfenyev, S. V. Filatov, M. Yu. Brazhnikov, S. S. Vergeles, A. A. Levchenko, "Formation and decay of eddy currents generated by crossed surface waves", arxiv.org/abs/1905.01875.

7. Model Verification

The surface film is also known to change the ratio of the amplitudes of horizontal $||u_\alpha||$ and vertical $||u_z||$ velocities on the fluid surface for the wave motion: $||u_\alpha||/||u_z|| = 1/\sqrt{\varepsilon^2-\varepsilon\sqrt{2}+1}$, and it also increases the wave attenuation rate $1/\tau_{th}$ after the wave excitation is turned off:

$$\frac{1}{\omega\tau_{th}} = 2\gamma^2\left(1+\frac{1}{\gamma kL\sqrt{2}}\right) + \frac{\gamma}{2\sqrt{2}}\frac{\varepsilon^2}{(\varepsilon^2-\varepsilon\sqrt{2}+1)}.$$



(a) The comparison of the ratio of horizontal and vertical velocities on the fluid surface for the wave motion measured directly (vertical axis) and calculated theoretically (horizontal axis). (b) The comparison of the wave decay time measured directly after the pumping is switched off (vertical axis) and calculated theoretically (horizontal axis).

8. Conclusion

- We discovered experimentally the Eulerian mean flow generated by nonlinear interaction of crossed surface waves and developed a theory explaining its excitation.
- The surface film is capable of significantly influencing the fluid flow in the bulk and it has a potential to control the intensity of generated currents in a wide range.

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